

Signaling and the Black Hole Final State

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In an attempt to restore the unitarity of the evaporation process, Horowitz and Maldacena [1] recently proposed a boundary-condition constraint for the final quantum state of an evaporating black hole at its singularity. Gottesman and Preskill [2] have argued that the proposed constraint must lead to nonlinear evolution of the initial (collapsing) quantum state. Here we show that in fact this evolution allows signaling, making it detectable outside the event horizon with entangled-probe experiments of the kind we proposed recently [3]. As a result the Horowitz-Maldacena proposal may be subject to terrestrial tests.

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We begin with a brief review of the final-state boundary condition for evaporating black holes as proposed in [1] and further elucidated in [2]. In the semiclassical approximation, the overall Hilbert space for the evaporation process can be treated as a decomposition

$$\mathcal{H} = \mathcal{H}_M \otimes \mathcal{H}_F = \mathcal{H}_M \otimes \mathcal{H}_{\text{in}} \otimes \mathcal{H}_{\text{out}}, \quad (1)$$

where \mathcal{H}_M denotes the Hilbert space of the quantum field that constitutes the collapsing body, and \mathcal{H}_F is the Hilbert space in which the quantum-field fluctuations around the background spacetime determined by the \mathcal{H}_M quantum state live. The separation of \mathcal{H} into \mathcal{H}_M and \mathcal{H}_F reflects the semiclassical nature of the treatment in a fundamental way. Moreover, the fluctuation Hilbert space \mathcal{H}_F can be further decomposed as $\mathcal{H}_F = \mathcal{H}_{\text{in}} \otimes \mathcal{H}_{\text{out}}$, where \mathcal{H}_{in} and \mathcal{H}_{out} denote the Hilbert spaces of fluctuation modes confined inside and outside the event horizon, respectively. Before evaporation, the quantum state $|\rangle \in \mathcal{H}$ of the complete system can be written as a product

$$|\rangle = |\psi_0\rangle_M \otimes |0_U\rangle, \quad (2)$$

where $|\psi_0\rangle_M \in \mathcal{H}_M$ is the initial wave function of the collapsing matter, and the Unruh vacuum $|0_U\rangle \in \mathcal{H}_F = \mathcal{H}_{\text{in}} \otimes \mathcal{H}_{\text{out}}$ is the maximally entangled state

$$|0_U\rangle = \frac{1}{\sqrt{N}} \sum_{k=1}^N |k_{\text{in}}\rangle \otimes |k_{\text{out}}\rangle. \quad (3)$$

Here N is the common dimension (the number of degrees of freedom necessary to completely describe the internal state of the black hole) of all three Hilbert spaces \mathcal{H}_M , \mathcal{H}_{in} , and \mathcal{H}_{out} , and $\{|k_{\text{in}}\rangle\}$ and $\{|k_{\text{out}}\rangle\}$, $k = 1, 2, \dots, N$, are fixed orthonormal bases for \mathcal{H}_{in} and \mathcal{H}_{out} , respectively. After the hole evaporates completely, the “final” Hilbert space is simply \mathcal{H}_{out} , and the usual semiclassical arguments inevitably imply a mixed state ρ_{out} as the

endpoint of complete evaporation (see Fig. 1 in [3] and the associated discussion), revealing that the transition $|\psi_0\rangle_M \mapsto \rho_{\text{out}}$ is manifestly non-unitary.

The Horowitz-Maldacena proposal (HM) imposes a boundary condition on the final quantum state at the black-hole singularity by demanding that it be equal to

$$|\Phi\rangle \equiv U^\dagger \left[\frac{1}{\sqrt{N}} \sum_{j=1}^N |j_M\rangle \otimes |j_{\text{in}}\rangle \right] \in \mathcal{H}_M \otimes \mathcal{H}_{\text{in}} \quad (4)$$

where $\{|j_M\rangle\}$ is an orthonormal basis for \mathcal{H}_M , and $U : \mathcal{H}_M \otimes \mathcal{H}_{\text{in}} \rightarrow \mathcal{H}_M \otimes \mathcal{H}_{\text{in}}$ is a unitary transformation. More precisely, HM states the following:

There exists a unitary map $U : \mathcal{H}_M \otimes \mathcal{H}_{\text{in}} \rightarrow \mathcal{H}_M \otimes \mathcal{H}_{\text{in}}$ such that with $|\Phi\rangle \in \mathcal{H}$ defined as in Eq. (4), the state $|\rangle$ [Eqs. (2)–(3)] evolves after complete evaporation as

$$|\rangle \mapsto \alpha P_{|\Phi\rangle \otimes \mathcal{H}_{\text{out}}} |\rangle, \quad (5)$$

where $\alpha \in \mathbb{R}$ is a renormalization constant, and $P_{|\Phi\rangle \otimes \mathcal{H}_{\text{out}}}$ denotes the projection onto the linear subspace $|\Phi\rangle \otimes \mathcal{H}_{\text{out}} \equiv \{|\Phi\rangle \otimes |v\rangle : |v\rangle \in \mathcal{H}_{\text{out}}\}$ of \mathcal{H} .

The unitary operator U describes the non-local evolution of the black-hole quantum state near the singularity, as well as its evolution in the semiclassical regime before the singularity; one would expect a full quantum theory of gravity to be able to completely specify this operator. To restore unitarity to the transition map $\mathcal{H}_M \rightarrow \mathcal{H}_{\text{out}}$, Horowitz and Maldacena [1] further demand that U be in the form of a product corresponding to the absence of entangling interactions between \mathcal{H}_M and \mathcal{H}_{in} :

$$U = S_1 \otimes S_2, \quad (6)$$

where $S_1 : \mathcal{H}_M \rightarrow \mathcal{H}_M$ and $S_2 : \mathcal{H}_{\text{in}} \rightarrow \mathcal{H}_{\text{in}}$ are unitary maps. To find the effective evolution map $\mathcal{H}_M \rightarrow \mathcal{H}_{\text{out}}$ resulting from HM and the assumption Eq. (6), start from the equality

$$\alpha P_{|\Phi\rangle \otimes \mathcal{H}_{\text{out}}} (|\psi_0\rangle_M \otimes |0_U\rangle) = |\Phi\rangle \otimes |X_{\text{out}}\rangle, \quad (7)$$

where $|X_{\text{out}}\rangle$ is the state in \mathcal{H}_{out} into which the initial state $|\psi_0\rangle_M$ evolves after the evaporation. Contracting

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both sides of Eq. (7) with $\langle \Phi |$ substituted from Eq. (4)

$$\begin{aligned} |X_{\text{out}}\rangle &= \frac{\alpha}{N} \sum_{j=1}^N \langle j_M | \otimes \langle j_{\text{in}} | (S_1 \otimes S_2) \\ &\quad |\psi_0\rangle_M \otimes \sum_{k=1}^N |k_{\text{in}}\rangle \otimes |k_{\text{out}}\rangle \\ &= \frac{\alpha}{N} \sum_{j=1}^N \sum_{k=1}^N \langle j_M | S_1 | \psi_0 \rangle_M \langle j_{\text{in}} | S_2 | k_{\text{in}} \rangle |k_{\text{out}}\rangle. \end{aligned} \quad (8)$$

In terms of the basis components $X_{\text{out } j} \equiv \langle j_{\text{out}} | X_{\text{out}} \rangle$, $\psi_{0 \ k} \equiv \langle k_M | \psi_0 \rangle_M$, $S_{1 \ jk} \equiv \langle j_M | S_1 | k_M \rangle$, and $S_{2 \ jk} \equiv \langle j_{\text{in}} | S_2 | k_{\text{in}} \rangle$, Eq. (8) can be rewritten in the matrix form

$$X_{\text{out } k} = \frac{\alpha}{N} \sum_{l=1}^N (S_2^T S_1)_{kl} \psi_{0 \ l}, \quad (9)$$

where S^T denotes matrix transpose of S . Since the transpose of a unitary matrix is still unitary, Eq. (9) shows that (i) the renormalization constant $\alpha = N$, and (ii) the transformation $|\psi_0\rangle_M \mapsto |X_{\text{out}}\rangle$ is unitary.

However, as pointed out by Gottesman and Preskill [2], entangling interactions between \mathcal{H}_M and \mathcal{H}_{in} are unavoidable in any reasonably generic gravitational collapse scenario. Consequently, we cannot expect the unitary operator U to have the product form Eq. (6) in general. For a general unitary map $U : \mathcal{H}_M \otimes \mathcal{H}_{\text{in}} \rightarrow \mathcal{H}_M \otimes \mathcal{H}_{\text{in}}$, the vector $|\Phi\rangle$ defined by Eq. (4) is an arbitrary element in $\mathcal{H}_M \otimes \mathcal{H}_{\text{in}}$, and Eq. (8) leads to the more general linear expression

$$X_{\text{out } k} = \alpha \sum_{l=1}^N T_{kl} \psi_{0 \ l} \quad (10)$$

instead of Eq. (9). Here T denotes the matrix

$$T_{kl} \equiv \frac{1}{\sqrt{N}} \langle \Phi | l_M \rangle \otimes |k_{\text{in}}\rangle \quad (11)$$

which is unconstrained except for $\sum_{kl} |T_{kl}|^2 = 1/N$. Only when U has the product form Eq. (6) T equals $(1/N \text{ times})$ a unitary matrix [Eq. (9)]. Note that the constant α is to be determined from the condition that $|X_{\text{out}}\rangle$ remains normalized. After this renormalization, we can express the transformation $\mathcal{H}_M \rightarrow \mathcal{H}_{\text{out}}$ described by Eq. (10) more succinctly in the form

$$X_{\text{out } k} = \frac{1}{(\sum_i |\sum_j T_{ij} \psi_{0 \ j}|^2)^{\frac{1}{2}}} \sum_{l=1}^N T_{kl} \psi_{0 \ l} \quad (12)$$

where now T is a *completely unconstrained*, arbitrary matrix [4]. While it maps pure states to pure states, the transformation $\mathcal{H}_M \rightarrow \mathcal{H}_{\text{out}}$ specified by Eq. (12) is not only nonunitary, but it is in fact *nonlinear*; linearity is recovered (along with unitarity) only when T is proportional to a unitary matrix.

In a recent paper [3], we argued that nonlinear quantum evolution inside an evaporating black hole might

have observable consequences outside the event horizon when an entangled system (whose coherence is carefully monitored) partially falls into the hole. We also proposed a specific experiment that should be able to detect the presence of such signaling nonlinear maps via terrestrial quantum interferometry. We will now show that the HM-class of nonlinear maps defined in Eq. (12) in fact belong to this signaling class. Therefore, the HM boundary-condition proposal can in principle be tested by terrestrial experiments.

Let us assume a causal configuration as depicted in Fig. 1 of [3], where a bipartite system AB evolves to send its B -half into a black-hole event horizon along a null geodesic, while the A -half remains coherently monitored outside the horizon. We can then further decompose the “collapsing” Hilbert space \mathcal{H}_M in the form $\mathcal{H}_M = \mathcal{H}_A \otimes \mathcal{H}_B$, where \mathcal{H}_B now corresponds to all matter that falls into the black hole, including the “probe beam” e of our trans-horizon Bell-correlation experiment (cf. Fig. 2 and the discussion following it in [3]), and \mathcal{H}_A corresponds to all matter that remains outside the horizon, including the interferometer beams which are monitored in the laboratory. We also identify the outgoing Hilbert space \mathcal{H}_{out} with \mathcal{H}_M , which amounts to specifying a unitary map $U_M : \mathcal{H}_M \rightarrow \mathcal{H}_{\text{out}}$ connecting orthonormal basis sets in the two spaces. With this identification, the “evaporation” map $\mathcal{H}_M \rightarrow \mathcal{H}_{\text{out}}$ can be treated as a map sending \mathcal{H}_M onto \mathcal{H}_M . Reinterpreted thus, the action of a general quantum map in the class defined by Eq. (12) can be written as

$$\rho_{AB} \mapsto \frac{T \rho_{AB} T^\dagger}{\text{Tr}(T \rho_{AB} T^\dagger)} \quad (13)$$

on any state ρ_{AB} in $\mathcal{H}_M = \mathcal{H}_A \otimes \mathcal{H}_B$, where $T : \mathcal{H}_A \otimes \mathcal{H}_B \rightarrow \mathcal{H}_A \otimes \mathcal{H}_B$ is a (nonsingular) general linear transformation [4]. To satisfy the locality condition as formulated in Eq. (14) of [3], the map T must have the product form

$$T = T_A \otimes T_B, \quad (14)$$

where $T_A : \mathcal{H}_A \rightarrow \mathcal{H}_A$ and $T_B : \mathcal{H}_B \rightarrow \mathcal{H}_B$ are general linear maps. Since subsystem A remains outside the event horizon, the evolution map T_A must remain unitary, and we can assume (for simplicity and without loss of generality) that $T_A = \mathbb{I}_A$. Then the quantum evolution map Eq. (13) acting on the Hilbert space $\mathcal{H}_M = \mathcal{H}_A \otimes \mathcal{H}_B$ takes the more transparent form

$$\mathcal{E}_{AB} : \rho_{AB} \mapsto \frac{\mathbf{1}_A \otimes \mathcal{T}_B(\rho_{AB})}{\text{Tr}[\mathbf{1}_A \otimes \mathcal{T}_B(\rho_{AB})]}, \quad (15)$$

where $\mathbf{1}_A = \mathcal{E}_A$ denotes the identity map on states of \mathcal{H}_A , and \mathcal{T}_B denotes the linear transformation (not a quantum map)

$$\mathcal{T}_B : \rho_B \mapsto T_B \rho_B T_B^\dagger \quad (16)$$

on states of \mathcal{H}_B . When ρ_{AB} is a product state $\rho_{AB} = \rho_A \otimes \rho_B$, the action of \mathcal{E}_{AB} has the manifestly local form

$$\mathcal{E}_{AB}(\rho_{AB}) = \mathcal{E}_A(\rho_A) \otimes \mathcal{E}_B(\rho_B), \quad (17)$$

where $\mathcal{E}_A = \mathbf{1}_A$, and \mathcal{E}_B is the nonlinear quantum map

$$\mathcal{E}_B : \rho_B \mapsto \frac{\mathcal{T}_B(\rho_B)}{\text{Tr}_B[\mathcal{T}_B(\rho_B)]} = \frac{T_B \rho_B T_B^\dagger}{\text{Tr}_B(T_B \rho_B T_B^\dagger)} \quad (18)$$

mapping \mathcal{H}_B -states onto \mathcal{H}_B -states (compare Eq. (17) above with Eq. (14) of [3]). By contrast, when ρ_{AB} is entangled the action of \mathcal{E}_{AB} does not have the simple product form of Eq. (17).

The criterion for a quantum map \mathcal{E}_{AB} to be signaling is identified in [3] (see Eq. (15) of [3] and the associated discussion there) as simply the condition that

$$\text{Tr}_B[\mathcal{E}_{AB}(\rho_{AB})] \neq \mathcal{E}_A[\text{Tr}_B(\rho_{AB})] \quad (19)$$

for *some* (necessarily entangled) state ρ_{AB} . Now consider a class of entangled states ρ_{AB} in the form of a convex linear combination

$$\rho_{AB} = \lambda_1 \rho_A \otimes \rho_B + \lambda_2 \sigma_A \otimes \sigma_B, \quad (20)$$

where ρ_A , σ_A , and ρ_B , σ_B are (normalized) states in \mathcal{H}_A and \mathcal{H}_B , respectively, and $\lambda_1 > 0$, $\lambda_2 > 0$, $\lambda_1 + \lambda_2 = 1$ are real coefficients. Introduce the real numbers

$$\begin{aligned} n_1 &\equiv \text{Tr}_B[\mathcal{T}_B(\rho_B)] = \text{Tr}_B(T_B \rho_B T_B^\dagger), \\ n_2 &\equiv \text{Tr}_B[\mathcal{T}_B(\sigma_B)] = \text{Tr}_B(T_B \sigma_B T_B^\dagger). \end{aligned} \quad (21)$$

The right-hand-side of Eq. (19) is simply $\text{Tr}_B(\rho_{AB})$ (recall that $\mathcal{E}_A = \mathbf{1}_A$):

$$\mathcal{E}_A[\text{Tr}_B(\rho_{AB})] = \lambda_1 \rho_A + \lambda_2 \sigma_A, \quad (22)$$

while the left-hand-side is

$$\text{Tr}_B[\mathcal{E}_{AB}(\rho_{AB})] = \frac{\lambda_1 n_1 \rho_A + \lambda_2 n_2 \sigma_A}{\lambda_1 n_1 + \lambda_2 n_2}. \quad (23)$$

But

$$\lambda_1 \rho_A + \lambda_2 \sigma_A \neq \frac{\lambda_1 n_1 \rho_A + \lambda_2 n_2 \sigma_A}{\lambda_1 n_1 + \lambda_2 n_2} \quad (24)$$

unless at least one of the conditions: (i) $n_1 = n_2$, or (ii) $\rho_A = \sigma_A$ holds. The condition (i) does not hold in general unless the linear operator T_B is unitary (or a scalar multiple of a unitary operator), and condition (ii) does not hold in general unless ρ_{AB} is a product state. Therefore the nonlinear quantum map \mathcal{E}_{AB} defined by Eqs. (15)–(16) is in general in the signaling class.

In summary, we have shown that the quantum maps which likely characterize quantum evolution through evaporating black holes according to the Horowitz-Maldacena [1] boundary-condition proposal are a signaling class. It is clear that the HM-class of maps are detectable with the same kind of apparatus we described previously, namely the Zou-Wang-Mandel (ZWM) interferometer depicted in Fig. 2 of [3] [see Eqs. (11)–(13) of [3] for a specific example of the detection signal likely to arise from a HM-class nonlinear map, in this case a 45° shift in the detector's interference fringes]. On the other hand, the precise nature of the signal produced in the ZWM interferometer when the probe beam is sent into an evaporating hole will depend on the nature of the unitary operator U characterizing the HM boundary condition, Eq. (4). If, as predicted [1, 2], the operator U involves nonlocal phases which oscillate chaotically at Planckian frequencies near the singularity, then each ZWM photon entering the hole is likely to experience a different nonlinear evolution map \mathcal{E}_{AB} , and the observed signal will be an average over such maps. It appears plausible that this averaging will affect the local interference pattern back in the laboratory by erasing relative phases and thus diminishing fringe visibility. A detailed discussion of this and other experimental questions will be found in a forthcoming paper [5].

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- [1] G. T. Horowitz and J. Maldacena, [hep-th/0310281](#).
 - [2] D. Gottesman and J. Preskill, [hep-th/0311269](#).
 - [3] U. Yurtsever and G. Hockney, [quant-ph/0312160](#).
 - [4] The only constraint on the linear operator T entering the construction Eq. (12) or Eq. (13) is that T must not have

- zero as an eigenvalue (e.g., projection-like operators are excluded), since with a singular T this construction fails to give a well-defined quantum map.
- [5] G. Hockney and U. Yurtsever, in preparation.